Consonance to Complexity | Exploring the Origins and Evolution of Musical Harmony

Silence, or the lack of noise, was the first definable sonic characteristic to permeate our universe. Then through quantum fluctuation, or another form of unpredictable spark, a single vibration reverberated through the void. From it arose the seeds of order and chaos, harmony and dissonance. Music was not an implicit creation by humans, but rather a product of the very force that created us. Each culture came to discover music in its own way, shaping it as a reflection of their environment. Universally, music has the capability to communicate in ways which words cannot. Whales pulsate cries beneath the waves, birds vocalize melodies from the skies, and humans innovate ways of manipulating sound to their will; with the hope of transmitting emotion and thought. It seems that music is a naturally occurring phenomenon that we have learned to wield, similar to gravity or fire.

Consonance and dissonance are opposing characteristics used to define one's subjective interpretation of musical harmony. One ear might perceive the jagged edges of a complex jazz chord as beautiful while the other might feel discomfort. Consonance and dissonance are not fixed truths, but rather acquired thresholds, formed by our natural adeptness to probe and our nurtured musical memories. The ancient drones of Gregorian chants for example required little to no prerequisite to be perceived as consonant; experimental electronic music however lies at a higher threshold of subjective consonance, hence it stands further from the core mathematical framework music is built around.

What is it about these sounds–these air molecules vibrating at a specific rate–that evoke such primal feelings within us? Why does consonance stimulate us while dissonance stirs unsettling discomfort?

Aldous Huxley has a gracefully insightful quote about what makes music so uniquely special: *"Music "says" things about the world, but in specifically musical terms. Any attempt to reproduce these musical statements "in our own words" is necessarily doomed to failure. We cannot isolate the truth contained in a piece of music; for it is a beauty-truth and inseparable from its partner. The best we can do is to indicate in the most general terms the nature of the musical beauty-truth under consideration and to refer curious truth-seekers to the original."*

Perhaps the answer lies in the mathematical truths underlying our musical constraints. Defined as "the practical science which studies the structural functioning of music" [1], music theory is a term commonly recognized but vaguely understood. Beneath the surface lies a complex, arguably divine architecture of integer relationships that reflect universal truths, unbound to man. This paper focuses on the mathematical birthing blocks of harmony and explores how the historical progression of consonance and dissonance can be understood through entropy.

As a reader, you must first understand the key Pythagorean musical concepts. This ancient knowledge is the practical foundation upon which all musical harmony is built. Pythagoras made the simple yet revolutionary discovery that harmonious sounds occur in the presence of whole number ratios:

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[1] van Dillen, Oscar. "Music Theory." Music, Writings, Projects and Art, 2024, www.oscarvandillen.com/tools_for_students/music_theory.

❖ Perfect Unison: 1:1

❖ The Octave - 2:1

❖ The Perfect Fifth - 3:2

❖ The Perfect Fourth - 4:3

These are essentially the simplest spaces between frequencies (interpreted as sound) that seem intuitively pleasing to the human ear.

Folklore has it that around 600 B.C., Pythagoras was walking through his village when the sound of a blacksmith's hammer caught his attention.[2] As he listened to the hammer striking the anvil, he couldn't help but notice unique "tones" like those of a song. He went and talked to the blacksmith, and after some fiddling around, they realized something. One hammer with exactly twice the mass of another would produce the same sound, but lower. This was the mathematical discovery of the octave (2:1), and as he played around with different mass ratios of hammers, he was also able to discover the perfect fifth (3:2) and the perfect fourth (4:3). He then tried to apply these ratios to musical strings and got the same result. These whole number ratios were universally true, independent of human interpretation.

The ratios can be understood through exponentiation (the use of exponents):

❖ Perfect Unison = 1^n (where n has no effect)

❖ The Octave = 2^n (where n represents how many octaves you move)

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[2] "The Fable of the Fifth Hammer & Diversity." Brand Fuel, 15 Aug. 2022, www.brandfuel.com/the-fable-of-the-fifth-hammer-diversity/.

- ❖ The Perfect Fifth = 3^n (where n represents how many perfect fifths you move)

- ❖ The Perfect Fourth = $\dfrac{2^n}{3^n}$ (where n represents how many perfect fourths you move)

This leads us to our next concept, a geometric symbol discovered by the Greeks that is said to carry divine philosophical and mathematical information: The Pythagorean Tetractys. It has appeared in many different forms throughout history, but traditionally the tetractys is an arrangement of ten points in the shape of a triangle (see Figure 1 for reference). Each row can be characterized as follows:

- ❖ 1st row: Monad *(Unity)*

- ❖ 2nd row: Dyad *(Duality)*

- ❖ 3rd row: Triad *(Harmony)*
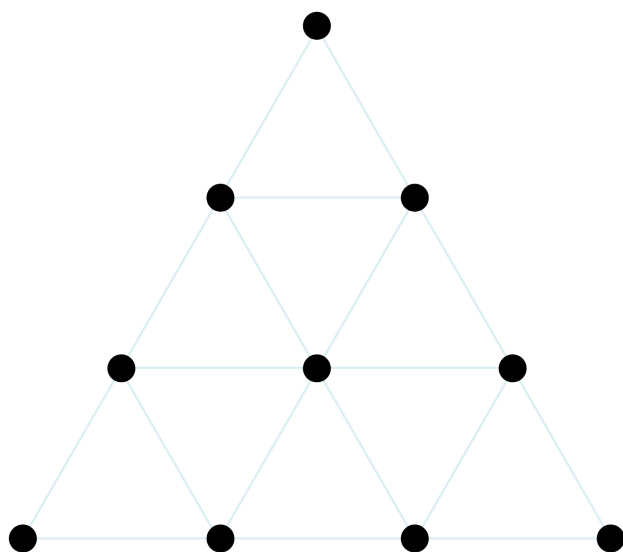
- ❖ 4th row: Tetrad *(Balance)*

Figure 1, *The Tetractys, Sterling Publishing, Public Domain*

The sum of these numbers $1 + 2 + 3 + 4 = 10$ is known as the **Dekad**.

Pythagoreans revered the Dekad as a symbol of completion, referring to it as "τέλειος ἀριθμός"
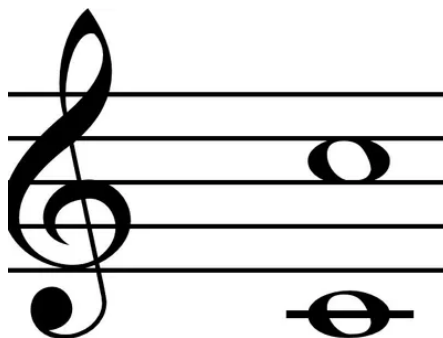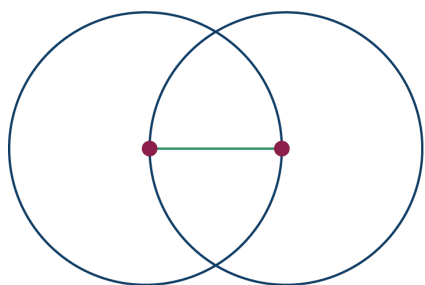
(perfect number)

By relating this back to the interval ratios mentioned earlier, the reflection of harmony within the

tetractys becomes apparent:
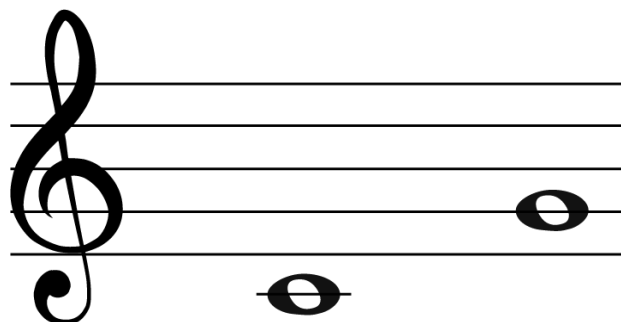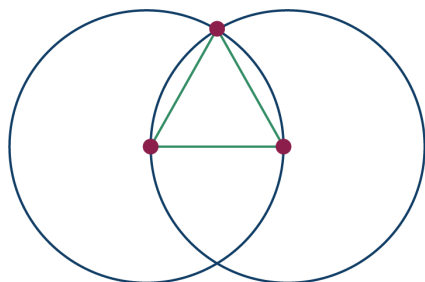
Monad = 1:1 (Perfect Unison)
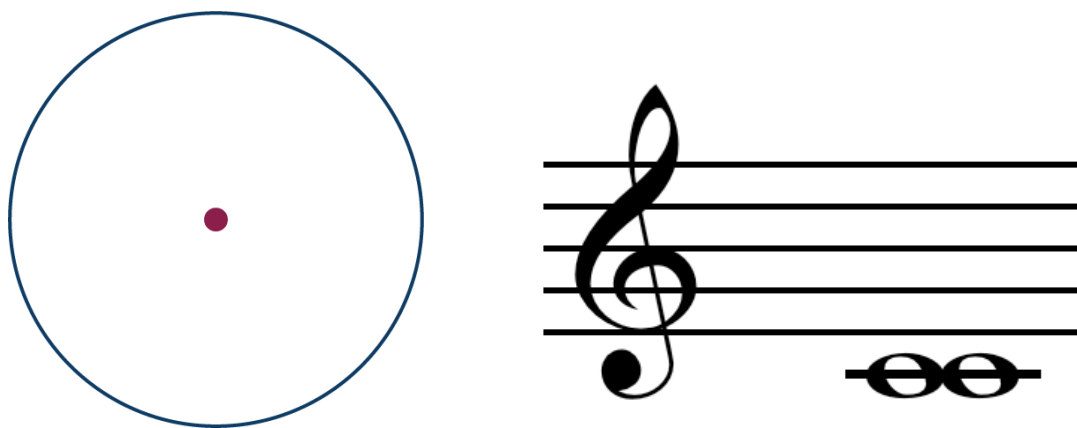
The Monad represents absolute undifferentiated oneness.
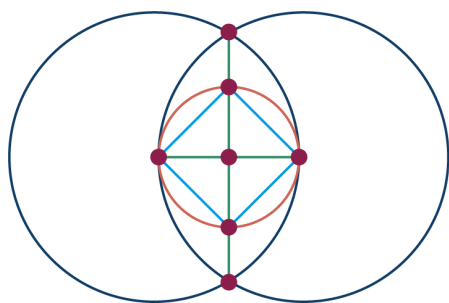
Dyad = 2:1 (Octave)

The Dyad is symbolic of division and attraction, it repels and attracts equally.

Triad = 3:2 (Perfect Fifth)

The Triad characterizes harmony, balance and strength.
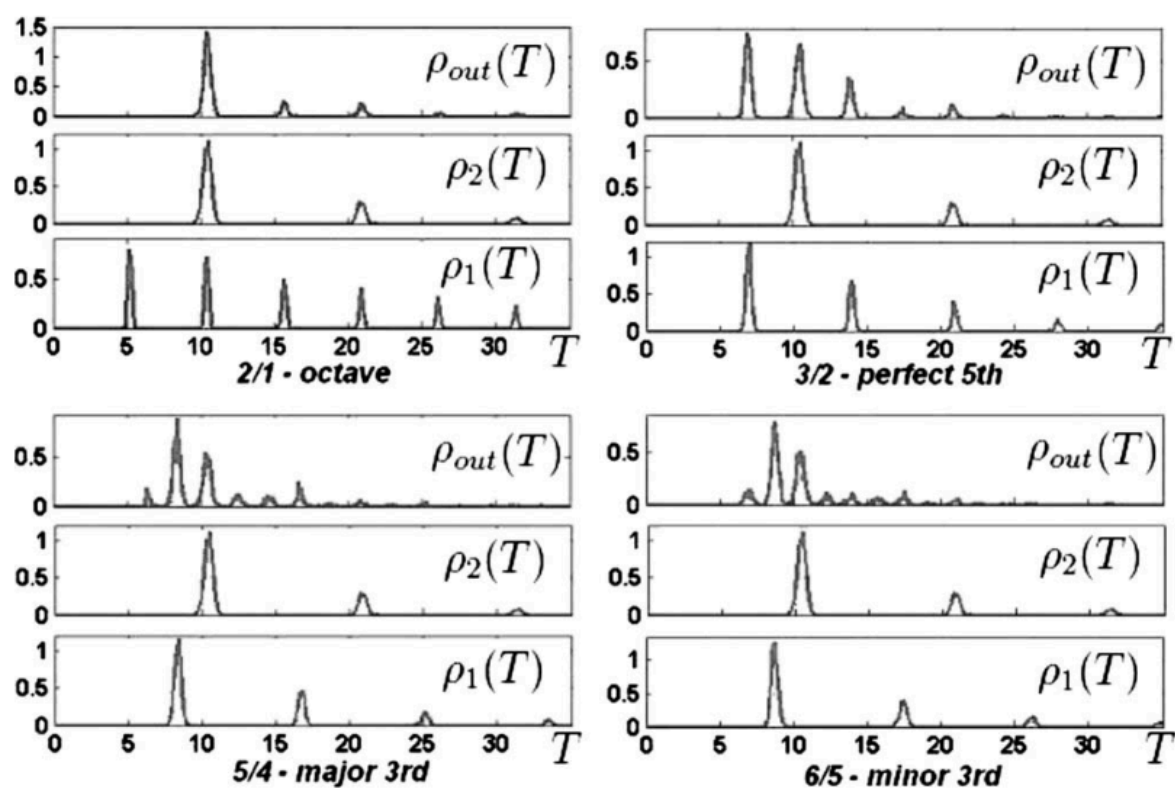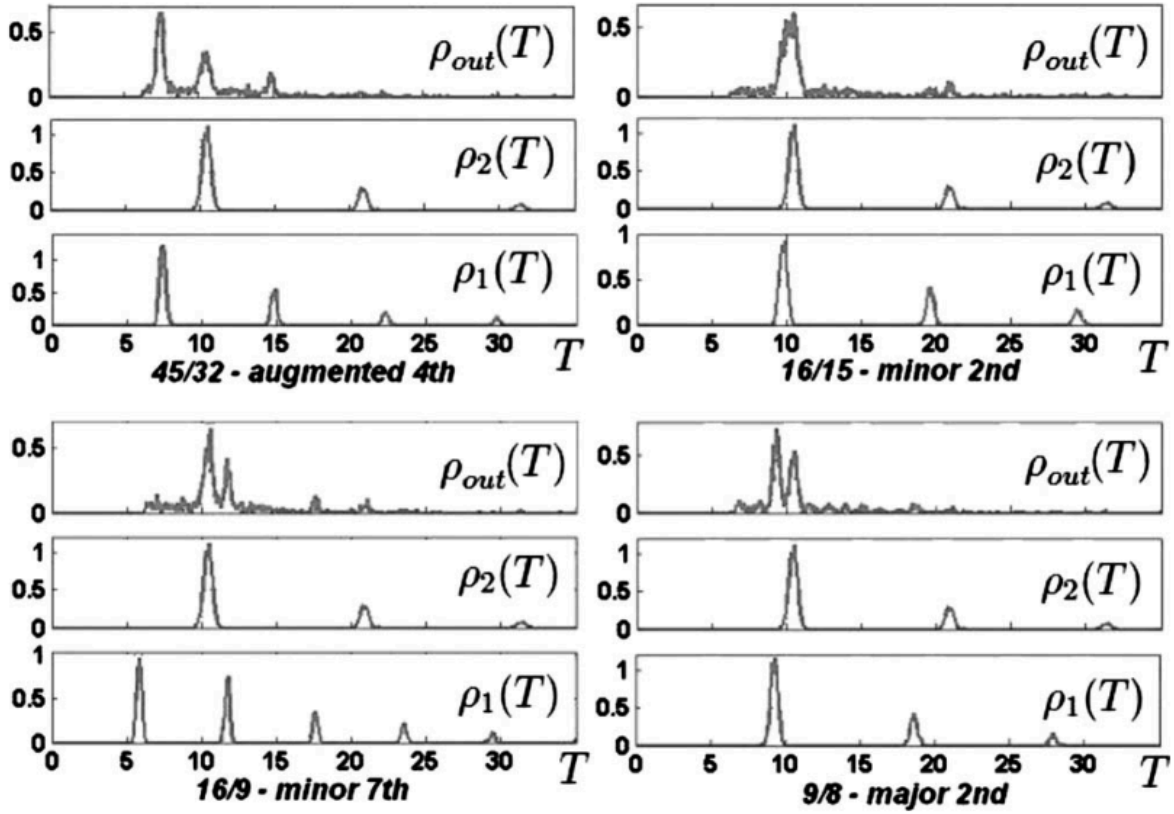
Tetrad = 4:3 (Perfect Fourth)

The Tetrad embodies completion and wholeness.

This ancient knowledge is important because we will now relate it to a modern understanding of harmony through the concept of entropy. Entropy broadly refers to the measure of how orderly or chaotic a system is. This essay will refer to entropy in the informational sense. Claude Shannon was the pioneer of informational entropy, creating the mathematical framework that enabled a groundbreaking understanding of uncertainty and randomness. His work was the catalyst for the development of the communications networks used around the world today. He introduced the "bit", which refers to a binary digit that can be either zero or one. This is the smallest, most basic unit of information that virtually all digital systems use.

With this in mind, we can begin to look at musical harmony through the lens of informational entropy. In 2011, researchers Ushakov, Dubkov, and Spagnolo measured how neurons in the brain respond to different interval ratios.[3] What they found was that more complex interval ratios resulted in a more irregular firing pattern of neurons with higher informational entropy. Meanwhile, the simplest ratios (as previously seen in the Tetractys) produced the lowest entropy signals.

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[3] Ushakov, Yury, et al. "Regularity of Spike Trains and Harmony Perception in a Model of the Auditory System." Research Gate, PubMed, Sept. 2011, www.researchgate.net/publication/51701145_Regularity_of_Spike_Trains_and_Harmony_Perception_in_a_Model_ of_the_Auditory_System.

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2/1 - octave

3/2 - perfect 5th

5/4 - major 3rd

6/5 - minor 3rd

$\rho_{out}(T)$ $\rho_2(T)$ $\rho_1(T)$

45/32 - augmented 4th   16/15 - minor 2nd

16/9 - minor 7th   9/8 - major 2nd

$Pout(T)$ in this case refers to the output distribution evoked by the two input frequencies $P1(T)$ and $P2(T)$. So for example:

When measuring the octave $P1(T)$ could correspond to 220 Hz (A3) and P2(T) might correspond to 440 Hz (A4). $Pout(T)$ then serves to represent the auditory response of both notes, which combined create the interval ratio.

The graphs above nicely illustrate the researchers' findings. If we look at how Pout(T) evolves as the interval ratios become more complex, it is visually apparent that the informational entropy increases.

From these results, it is reasonable to conclude that the interval ratios used to make harmony are part of a gradient or spectrum ranging from the lowest possible entropy (1:1 perfect unison) to potentially infinity. It makes sense that the first mathematical music system arose at the very bottom of this gradient (i.e., tetractys) and continues to be expanded to this day. We continue to push the limits, transforming previously perceived dissonance into consonance, and by embracing increasing complexity we are effectively "decoding" the entropy of music.

Now, while humanity appears to be on a general trajectory towards higher forms of musical entropy, this progression is not linear. Musical evolution occurs in periods of innovation, consolidation and regression. Classical music and jazz are periods that are particularly representative of this. During the classical period, composers focused on order, creating music that sounded balanced. The music was therefore very perceptively consonant with low entropy. It was an era of consolidation for harmony, focusing on elegant proportions. On the other hand, jazz emerged in the early 20th century and pushed the boundaries, leaping towards higher levels of entropy. The use of intervals previously considered dissonant (ninths, elevenths, thirteenths) redefined the musical experience in ways that were uncomfortable for many. Jazz demonstrated a level of musical entropy that could be coherently expressed in ways that had been thought impossible.

It is important to note that musical movements are somewhat subjective categorisations of genre, and even within the same category they can vary widely. Even during certain periods of innovation or consolidation, there will always be artists who produce against the grain. However, when looking at musical evolution from a macro perspective, it seems that each period of

innovation reaches new levels of complexity, and each period of consolidation returns to a baseline above the latter.

Music fills an irreplaceable artistic void that is simultaneously mathematical and metaphysical. Our journey has progressed through this art form from a simple Pythagorean Tetractys, to the complexities of today's musical landscape. This is illustrative of our broader progression towards understanding more chaos as order, more dissonance as consonance. We keep discovering new ways to transform discord into beauty, expanding our range of expression and understanding. So as we move forward on this trajectory, what lies beyond the horizon; is it a heart or the edge?

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